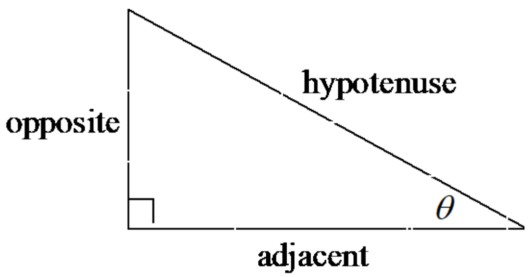
Trigonometry and Algebra Formulas

TRIGONOMETRY

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half Angle Formulas

$$\sin^2\theta = \frac{1}{2}\left(1-\cos(2\theta)\right)$$

$$\cos^2\theta = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

Sum and Difference Formulas

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\tan\beta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Inverse Trig Functions

$y = \sin^{-1}x$ is equivalent to $x = \sin y$

$y = \cos^{-1}x$ is equivalent to $x = \cos y$

$y = \tan^{-1}x$ is equivalent to $x = \tan y$

Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

ALGEBRA

Arithmetic Operations

$$ab + ac = a(b+c) \qquad a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc} \qquad \frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b}+\frac{c}{d} = \frac{ad+bc}{bd} \qquad \frac{a}{b}-\frac{c}{d} = \frac{ad-bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c} \qquad \frac{a+b}{c} = \frac{a}{c}+\frac{b}{c}$$

$$\frac{ab+ac}{a} = b+c,\ a\neq 0 \qquad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

The information for this handout was compiled from the following sources:

Paul's Online Math Notes. (n.d.). Retrieved from http://tutorial.math.lamar.edu/cheat_table.aspx

Trigonometry and Algebra Formulas

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Exponent Properties

$a^n a^m = a^{n+m}$ $\quad \dfrac{a^n}{a^m} = a^{n-m} = \dfrac{1}{a^{m-n}}$

$(a^n)^m = a^{nm}$ $\quad a^0 = 1, \ a \neq 0$

$(ab)^n = a^n b^n$ $\quad \left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$a^{-n} = \dfrac{1}{a^n}$ $\quad \dfrac{1}{a^{-n}} = a^n$

$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$ $\quad a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = \left(a^n\right)^{\frac{1}{m}}$

Logarithms and Log Properties

Definition

$y = \log_b x$ is equivalent to $x = b^y$

Factoring Formulas

$x^2 - a^2 = (x+a)(x-a)$

$x^2 + 2ax + a^2 = (x+a)^2$

$x^2 - 2ax + a^2 = (x-a)^2$

$x^2 + (a+b)x + ab = (x+a)(x+b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$

$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$

$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$

$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

Quadratic Formula

Solve $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Line/Linear Function

$y = mx + b$ or $f(x) = mx + b$

Graph is a line with point $(0, b)$ and slope *m*.

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$y = a(x-h)^2 + k \qquad f(x) = a(x-h)^2 + k$

Or

$y = ax^2 + bx + c \qquad f(x) = ax^2 + bx + c$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

Circle

$(x-h)^2 + (y-k)^2 = r^2$

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

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